# Engineering Notes

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## Frequency Response of Stationary Internal Wave Probes

H. E. Gilreath* and C. R. Walton†
*The Johns Hopkins University, Silver Spring, Md.*

STATIONARY single-electrode conductivity probes have been used a number of times in the past to make internal wave measurements in stratified liquids for which the density variation corresponds to a variation in specific conductance,[1-3] e.g., salinity stratification. In fluids with zero mean velocity, this specific implementation can be expected to result in a frequency response very much below the kilohertz range reported for similar probes used for measuring turbulent fluctuations in a streaming flow,[4] but since the frequencies of interest in internal wave studies are of the order of 0.1 Hz, this fact has not discouraged their use. Basically the response problem is due to the absence of a mean flow of sufficient strength to convect disturbed fluid away from the electrode rapidly. As a result the probe ends up measuring its own wake as the same disturbed flow oscillates past the tip before the fluid can completely recover. This problem, it should be noted, is distinct from that dealing directly with the rate of mass transfer at the tip, which involves the usual boundary-layer considerations.

Of Refs. 1–3, only Lofquist[2] gives a quantitative statement concerning the response of a stationary probe, which in terms of the ratio of indicated-to-true amplitude he reports to be no worse than 0.7 at frequencies near the local Vaisala frequency, $N = [(-g/\rho)d\rho/dz]^{1/2}$, where $g$ is the gravitational acceleration; $\rho$, the density; and $z$, the vertical coordinate. For typical laboratory tests, $N \simeq 0.2$ Hz. Although Lofquist's result, together with the sensitivity of available instrumentation, gave us confidence in choosing conductivity measurements as the central means for determining internal wave amplitudes in stratified flow studies, our initial test results suggested that a response problem existed. Its severity became apparent when dynamic calibration equipment was fabricated to replace the static calibration devices used earlier. Measurements over the pertinent frequency range and for amplitudes near those encountered in the experiments indicated a drastic fall-off in response at frequencies well below those of interest (Fig. 1). These results and a crude but fundamental analysis of the problem are presented here. Several conclusions are drawn about the response of stationary probes in general, and several possible solutions are discussed briefly.

### Dynamic Calibration Results

In Fig. 1 the open circles indicate the results for a conventional probe, i.e., a platinized Pt wire electrode encased in glass tubing drawn to approximately 0.050-in. diam. (Fig. 2a). The triangular symbols represent a test of a special wire probe (Fig. 2b) which was tried because it was believed at first that the poor response might somehow be due to the occurrence of extensive mixing in the vicinity of the probe tip induced by the probe body. This probe (akin to the type used by Lofquist) consisted of a 0.003-in.-diam. coated wire stretched horizontally between supporting tines. The coating was removed from a small length (~0.02 in.) near the center of the wire, which was then platinized to form the sensing electrode. In principle, the electrode is remote from the disturbances caused by the supports and its small diameter should assure a laminar wake. In practice, no improvement was noted (Fig. 1).

In Fig. 1 the amplitudes ($A$) are ratioed to the amplitude $A_0$ at the lowest frequency used in the testing, (i.e., 0.0042 Hz, corresponding to a period of 4 min/cycle). For practical purposes this is taken to represent the zero-frequency (static) result. The calibration sequence consisted of a sweep through a range of frequencies at a constant amplitude of 0.10 in. The calibration mechanism moved the probe smoothly in a vertical circular orbit to simulate the motion of an internal wave as simply as possible. For 10–20% variations of local $N$, the behavior shown in Fig. 1 was found to be independent of $N$.

Since probe design had no measurable effect, attention was shifted to the response of the fluid itself, which in turn motivated an attempt to operate the probe with a mean flow by mounting it on a horizontally translating stage. To check the response under these new circumstances, the probe was simultaneously oscillated vertically at a frequency of 0.1 Hz. The results in terms of the amplitude ratio as a function of translation speed are shown in Fig. 3. This mode of operation completely eliminates the "own-wake" mixing problem, and the figure indicates that good response is realized at reasonably low translational speeds. This is an important result because for studies of a propagating wave system the translational speed must be less than the component of the phase velocity in the direction of movement. Otherwise, the probe would soon outrun the waves. Clearly, in interpreting the data the output of the probe must be referred to a probe-fixed coordinate system, but this is a convenient circumstance for our own work, and translating probes are now routinely employed. As a dynamic calibration, the response of the moving probe to a sudden jump in vertical position (step-function) is monitored before each test. The resulting "time constant" is used to compute the amplitude response and phase shift by standard methods.

### Discussion

Recognizing that it is not always convenient, or even possible, to work with translating probes, the question remains: Under what conditions do stationary probes give acceptable results? In order to identify the basic parameters involved, a simple small-perturbation analysis of the problem was made. For small perturbations to both the velocity and salinity fields surrounding the probe, the (linear) diffusion equation, in probe-fixed coordinates, applies in the form

$$U_0 \partial s/\partial x = D\partial^2 s/\partial y^2 \qquad (1)$$

where $s$ is the salinity perturbation ($s = S_\infty - S$) and $D$ is

Received January 5, 1973. This work was supported under Navy Contract N-00017-72C-4401
Index category: Hydrodynamics.

*Senior Staff Engineer, Hypersonic Propulsion Group, Applied Physics Laboratory.
†Associate Engineer, Propulsion Research Facility, Applied Physics Laboratory.

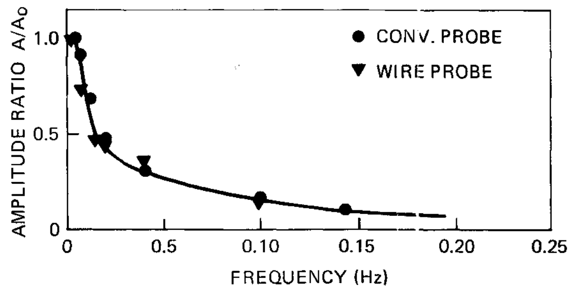


**Fig. 1 Stationary probe response.**

the diffusivity. A quasi-two-dimensional problem is formulated ($x$,$y$ being treated as curvilinear coordinates with sufficiently large radius of curvature), approximately corresponding to a wire probe whose length is much larger than its diameter and with the motion transverse to its axis. For a probe undergoing a vertical circular oscillation of amplitude $A$, $U_0$ in Eq. (1) is taken as $\pi A/T$, where $T$ is the period. (After a distance corresponding to $x = U_0T$ the probe "catches up" with its own wake.) If for simplicity an average initial perturbation ($s_0$) is assumed, (i.e., at $x = 0$, $s = s_0$ for $0 \leq y/d < ½$ and $s = 0$ for $y/d > ½$) the solution of Eq. (1) along the centerline in the wake can be shown to be

$$\frac{s}{s_0} \approx \frac{1}{2}\left[\frac{ReSc}{\pi(x/d)}\right]^{1/2} \tag{2}$$

where the small-argument approximation for the error function has been used ($Re = U_0d/\nu$ is the Reynolds' number, $Sc = \nu/D$ is the Schmidt number). Equation (2) shows that: 1) the initial perturbation relaxes with distance (or time) according to $x^{-1/2}$, and, since $ReSc \sim d$, the factor multiplying the relaxation distance for a given $s/s_0$ is inversely proportional to the probe diameter (smaller probes respond better); and, 2) if we arbitrarily choose a value of $s/s_0 = 0.1$ to characterize a relaxation time, $t_r$, (using $t = x/U_0$) we find from Eq. (2):

$$t_r = \tau^*/4\pi(s/s_0)^2 \approx 8\tau^* \tag{3}$$

where $\tau^* \equiv d^2/D$ is the characteristic relaxation time of the fluid associated with a disturbance of a size $0(d)$. For a probe with $d = 0.006$ in. in a dilute saline solution, $\tau^* \approx 20$ sec. A fundamental problem is thus recognized to stem from the very small diffusivity of saline solutions, leading to relaxation times of the order of minutes, while the phenomena under investigation usually have characteristic times of only several seconds.

So far, we have not addressed the specification of the initial disturbance $s_0$ nor the convective diffusion at the electrode tip itself. If $s_0$ is very small and the total change in salinity associated with one wave cycle is large, there is no serious response problem associated with the probe measuring its own wake. We expect that the magnitude of $s_0$ is related to the spatial scale of the disturbance ($s_0 \propto N^2d$), while the total change is related to the wave amplitude ($\Delta S \propto N^2A$). Therefore, using $U_0 = \pi A/T$, we find at the end of a cycle (i.e., $x = U_0T$)

$$s/\Delta S \propto (d/A)(\tau^*/T)^{1/2} \tag{4}$$

so that increasing the ratio of wave amplitude to probe diameter should improve the response. Moreover, the increased orbital velocities attending large amplitudes enhances the convective transfer at the tip. This offers a possible explanation for the excellent results of the large-amplitude (~1 in.) dynamic calibration tests of stationary probes performed in our laboratory, and may also explain Lofquist's calibration data, for which he did not give the

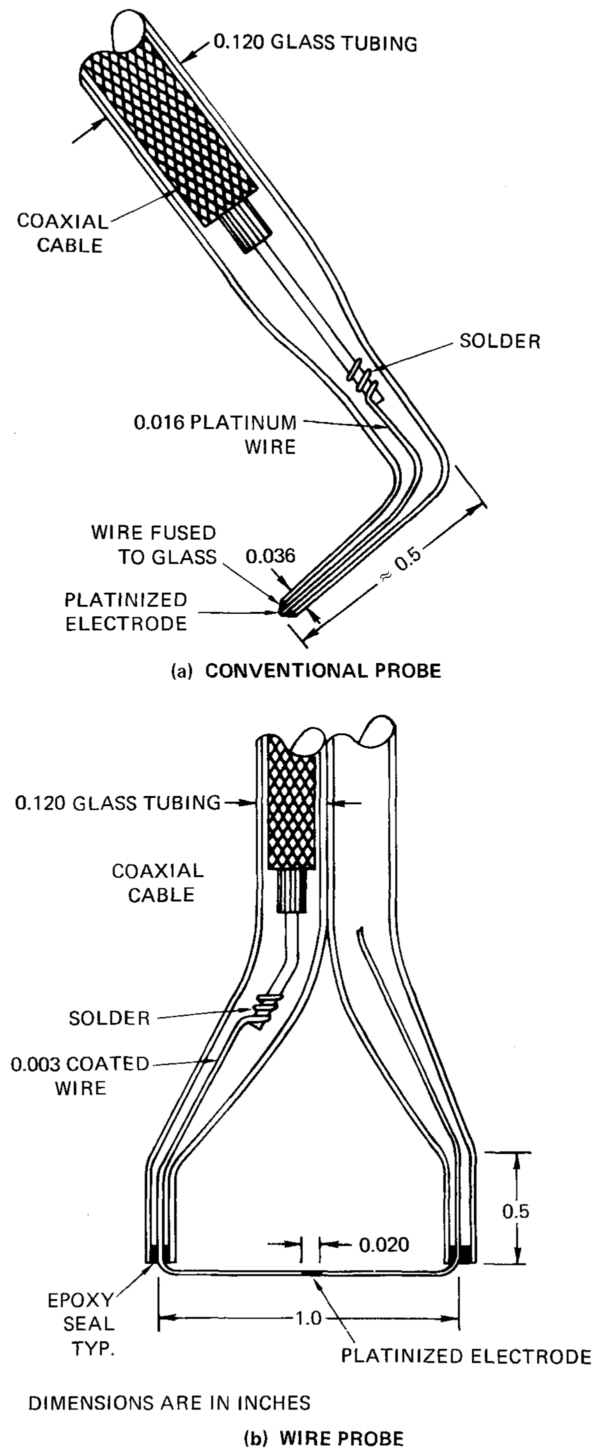


**Fig. 2 Schematic of conventional and wire probes.**

details. Unfortunately, in lab tests $A/d$ is often 0(1) or at most 0(10).

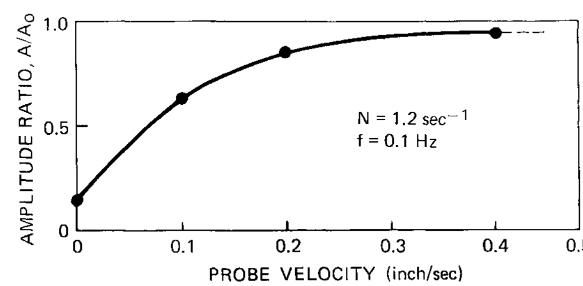


**Fig. 3 Translating probe response.**

## Conclusions

The preceding discussion was not expected to lead to an accurate quantitative assessment of the response problem associated with stationary probes, but it has revealed that:

1) For laboratory tests using saline solutions, the fundamental difficulty is the mismatch between the time constant of the fluid and the characteristic time of the experiments. The time constant $\tau^*$ can be reduced by decreasing the effective probe diameter, but the diffusivity is so low that disturbance dimensions of the order of 0.001 in. or less appear to be required before any sizeable improvement is realized from this approach. This is not considered a feasible solution at this point, and we must conclude that for most laboratory applications stationary probes are not suitable for measuring small-amplitude internal waves. However, for reasonably large amplitudes‡ Eq. (4) shows that the response can be improved by increasing the period $T$ and/or the amplitude $A$ of the internal waves generated during a test. Increasing $T$ may not be practical, however, since a weaker density gradient (leading to longer period waves) often introduces electronic difficulties with the instrumentation. Further, in a given scaled experiment, the ratio of wave amplitude-to-characteristic length is fixed, so that the capability for increasing $A$ is limited by the basic facility size.

2) The wake mixing portion of the response problem can be side-stepped, and the convective transfer at the electrode enhanced, by steadily translating the probe. The resulting data must be interpreted accordingly, but for many applications this should be feasible.

3) Nothing in the preceding analysis restricts the results to salinity stratifications or to laboratory facilities. The variable $s$ could be replaced by a temperature perturbation, and $D$ by the thermal diffusivity. Since the thermal diffusivity is roughly two orders of magnitude larger than the mass diffusivity, this means that: a) in a thermally stratified lab facility, $\tau^*$ is of the order of seconds rather than minutes, and the response problem is considerably alleviated (but must still be accounted for if the internal wave period is a few minutes or less); and b) in field measurements of internal waves, where "natural" stratifications are largely thermal, and (under the same circumstances) the ratios $d/A$ and $\tau^*/T$ in Eq. (4) are of the order of $10^{-3}$, response problems with stationary probes are not anticipated. Moreover, it is unlikely that such measurements would ever arise without an appreciable mean flow being present.

‡As a rough estimate, for dilute saline solutions we might use Fig. 3 to suggest amplitudes having corresponding orbital velocities of 0(0.5 in./sec), and further, amplitudes large enough to make the percentage error involved in the wake mixing effect small in accordance with Eq. (4).

## References

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